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NAVAL POSTGRADUATE SCHOOL

Monterey, California

TECHNICAL

TRANSFORMATION OF DISPLACEMENT

AND ORIENTATION CORRECTION DATA

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The simulation of an underwater vehicle track has been used to develop reference distributions for displacement and orientation corrections to the test range sensor arrays. These distributions show unusual behavior. Of particular concern are the scatter plots of the magnitude of displacement and orientation rotation angle, which exhibit a "hairpin" shape. The mechanisms producing this shape are interpreted and data transformations are developed that produce scatter plots of more regular form. In addition, some plotting methods are applied to provide some visual information about the joint distribution of the transformed data.

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I. Introduction

Imagine the scatter plot of data from two strongly correlated variables with the provision that only the magnitudes of the individual variables are recorded. Such data can produce "hairpin" shaped scatter plots (see Figure 1) and it would be useful to develop methodology for straightening the hairpin, i.e. for assigning negative signs to some of the values. Then perhaps one or more of the following could be achieved with the transformed data:

i. The scatter plots could be fitted with bivariate density functions, whose contours are convex sets.

ii. The scatter plots could be fitted with bivariate normal density functions.

In general this problem is a difficult one. Some, but not all, of the points that fall near a coordinate axis need to be reflected across that axis. A rule for making such selections should have some depth. The present work does not treat the general problem. We are more interested in expediency so that the general nature of the straightened scatter plot may be better understood. Also, our data structure contains concomitant information that will be exploited.

The mother project, of which this is a part, deals with the maintenance of calibrations of the 3-D underwater test ranges operated by NUWES. The tracking of vehicles is performed by a grid work of three dimensional sonar arrays located underwater at the bottom. A number of these individual arrays contribute tracking information which is integrated to reconstruct the path of the targeted vehicle. Mismatches occur when the splicing of individual contributions is done and the possibility that they are caused by sensor movement must be faced.

Fuller details can be found in References 1, 2, 3. Of immediate concern is the product of the program KEYMAIN which takes data from the array overlap region and estimates the relative displacement and orientation corrections that minimize the track splicing mismatch. To judge the significance of these estimates one needs a reference distribution which describes probabilistically the amount of variability that must be tolerated in a given situation. A simulation model, whose purpose is to generate such reference distributions, has been developed and is described in Reference 1. It simulates the path of the vehicle, complete with inherent or random variability, for each of the two sensor arrays in an overlap region. The results are passed to the program KEYMAIN, which estimates the corrections to one array relative to the other one.

The output of KEYMAIN contains a pair of three dimensional vectors. The first is the set of three displacement corrections (downrange, crossrange, and vertical) for an array's location. The second is the set of three Euler angles that describe the orientation correction for the array. Use is made of two other ways to describe these latter corrections:
(i) The orthonormal matrix B which provides the information in the form of a linear transformation. (ii) The axis direction numbers and the angle of rotation.

It is hoped that these six pieces of information can be replaced with two, namely the magnitude of the displacement and the angle of rotation. The joint distribution of these two can be economically plotted on a piece of paper, and it seems reasonable that this should be adequate to support the decision processes mentioned above. Such are the data in Figures 1 and 2.

Some additional unusual behavior is exhibited in Figure 2; namely, there seems to be a "base line of support" which the bivariate data do not cross. The mechanism that produces this is not understood at this writing, and this feature will not be treated.

The organization of the report is as follows: Section II contains the mathematical development of methodology to " straighten the hairpins" and perform ad hoc graphic tests for the bivariate normal distribution. Section III contains the application of this methodology to our six cases of simulated data. A summary follows.

II. Mathematical Support

The simulated data are six dimensional and are best viewed as two strongly correlated 3-D sets; the three components of displacement and the three components of orientation. As a data reduction technique let us draw down to two dimensions; the magnitude of displacement d, and the magnitude of the angle, Θ, of reorientation about an axis of rotation. It is the scatter of Θ on d that exhibits the unusual behavior (e.g. Figure 1).

The hairpin shape is readily interpreted once one is aware that displacement and orientation are strongly correlated, and that the 3-D data swarm of displacement is very long and narrow. That is, the variability of displacement is dominated by its first principal component. At one end of this principal component both Θ and d are large. They both become smaller as one moves away from the extreme and toward the center of the principal component. But one of these, say Θ, reaches its minimum first and starts becoming larger again before the other, d, achieves its minimum value. This may cause an observable "arch on the pin". Then the data fills out the other leg of the pin as we move to the other extreme of the principal component. Figures 3 and 4 bear this out. These are scatter plots of d, resp. Θ, on the first principal component, u. The key to straightening the hairpins may be found in assigning negative values to the left leg of these "v" shaped scatter plots.

This section contains the mathematical support needed to straighten the hairpin and perform an ad hoc test for bivariate normal distribution to the result. It contains three subsections: (i) Development of formulas relating the three ways to represent the orientation corrections. (ii) Discussion of methods to assign negative signs to some of the d and Θ. (iii) Development of an ad hoc test for bivariate normal distribution. They are presented in order.

Subsection (i). <u>Three Ways to Represent the Orientation Correction</u>

The three ways are:

1) Orthonormal matrix (B).

2) Euler angles (ϕ_1, ϕ_2, ϕ_3).

3) Axis (x) and rotation (Θ).

Each has its advantages. We proceed to describe them and show how to transform from one to the other.

1. An orthonormal matrix B is defined by the property $B^TB = I$ (the identity matrix) and may be viewed as a linear transformation that preserves distance. Thus, if x and y are two vectors, the distance separating them can be computed from

$$||x - y||^2 = (x - y)^T(x - y) = \Sigma(x_i - y_i)^2.$$

To illustrate the distance preserving property, let $x' = Bx$ and $y' = By$ represent the images of x and y under the linear transformation B. Then

$$||x' - y'||^2 = ||Bx - By||^2 = ||B(x - y)||^2 = (B\ (x - y))^T(B(x - y))$$

$$= (x - y)^T\ B^TB(x - y) = (x - y)^T(x - y) = ||x - y||^2$$

Thus the distance between x and y and the distance between their images is the same. These tranformations are the rigid rotations. Because of the constraint, $B^TB = I$, the matrix B has only three degrees of freedom. Three are lost because each column must have length unity, and three more are lost because each pair of columns must be orthogonal.

2. The three Euler angles specify the rotations that take place in the base planes of the coordinate system as each axis is held fixed. Since the positions of these planes change with each rotation, it is necessary to specify the successive order in which the axes are held fixed. We choose to first hold the x-axis (downrange) fixed and rotate the y - z plane through all angles of ϕ_1; then hold the y-axis (crossrange) fixed in its new position so as to rotate the (new) x - z plane through an angle ϕ_2; followed by holding the z-axis (vertical) fixed in its (twice) tranformed position and rotate the x - y plane through an angle ϕ_3.

In navigation work, these three angles are called, roll, pitch, and yaw. In the NUWES software they are designated as Y-TILT, X-TILT, and Z-ROT.

The Euler angles are easily related to the B-Matrix. Each is a rotation in a plane of two coordinate axes while the third is held fixed. It follows that each rotation has its own representation as an orthonormal matrix. Thus, letting

$$S_i = \sin(\phi_i) \qquad \text{and} \qquad C_i = \cos(\phi_i)$$

for i = 1, 2, 3, we can specify:

$$B_1 = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & C_1 & -S_1 \\ 0 & S_1 & C_1 \end{Bmatrix} \qquad B_2 = \begin{Bmatrix} C_2 & 0 & -S_2 \\ 0 & 1 & 0 \\ S_2 & 0 & C_2 \end{Bmatrix} \qquad B_3 = \begin{Bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

(Note that we have specified clockwise, or "back" rotation. Then, using the stated order of application,

$$B = B_3 B_2 B_1$$

and the algebraic result of these multiplications is

$$B = \begin{Bmatrix} C_2 C_3 & -(S_1 S_2 C_3 + C_1 S_3) & -(C_1 S_2 C_3 - S_1 S_3) \\ C_2 S_3 & -(S_1 S_2 S_3 - C_1 C_3) & -(C_1 S_2 S_3 + S_1 C_3) \\ S_2 & S_1 C_2 & C_1 C_2 \end{Bmatrix}$$

Given B, the Euler angles can be recovered in the following way. Compute principle value solution using

$$\sin(\phi_2) = b_{31} \quad \sin(\phi_1) = b_{32}/\cos(\phi_2) \quad \sin(\phi_3) = b_{21}/\cos(\phi_2)$$

where

$$-\pi/2 < \phi_i < \pi/2 \qquad \text{for} \qquad i = 1,2,3.$$

Then check for possible modification of ϕ_1 and ϕ_3 using the signs of b_{11} and b_{33}. That is, make replacements.

$$\phi_1 \leftarrow \pi - \phi_1 \text{ if } b_{33} < 0$$

$$\phi_3 \leftarrow \pi - \phi_3 \text{ if } b_{11} < 0$$

3. The axis and rotation representation gives recognition to the fact that a rigid turn in three space must involve the turning of the entire space through an angle of Θ about an axis x. The only points unmoved by this transformation are the points on the axis. Thus

$$Bx = x$$

and the axis x is identified as the eigenvector of B having eigenvalue one. In order to make it unique, let us agree that x is normalized so that $||x||^2 = 1$ and it lies in the half space defined by a non-negative first component, $x_1 \geqq 0$. Note there are two degrees of freedom in this x.

To determine the rotational angle Θ, let us set up a righthanded coordinate system with x as its first axis, and then determine the angle of rotation resulting from applying B to any vector orthogonal to x. Our goal is to construct a change of basis matrix C which has x as its first row. It remains to specify the second and third rows, y and z. Since x is pointed in the half space $x_1 > 0$, let us apply the Gram-Schmidt process to $e_2^T = (0,1,0)$ in order to specify y.

I.e.

$$y = e_2 - (x_1, e_2)x$$

followed by a normalization to give it length one. Thus

$$y^T = (-x_1 x_2,\ 1 - x_2^2,\ -x_2 x_3)/\sqrt{1 - x_2^2}$$

In the construction of z we want to be assured that the result is a right handed system, so we use the vector cross product.

$$z = x \times y = \det \begin{vmatrix} e_1 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

where the representation by the determinant is a schematic one; inferring an expansion by the first row, whose components are the unit coordinate vectors.

The result is

$$z^T = (-x_3,\ 0,\ x_1)/\sqrt{1 - x_2^2}$$

Let us apply our change of basis matrix C

$$C^T = (x, y, z)$$

to the vector By. From geometrical considerations, we have By rotates y through the angle Θ and its representation in the basis C must be

$$CBy = \begin{Bmatrix} 0 \\ \cos(\Theta) \\ \sin(\Theta) \end{Bmatrix}$$

The third component can be inverted to compute the angle θ. A check of the sign of the second component will tell us the quadrant to which θ belongs. Thus x and θ can be constructed from the orthonormal matrix B.

To reverse the process, i.e. to construct B from the axis x and angle θ we can proceed as follows:

First construct the orthonormal matrix C from x exactly as before. Then apply the matrix

$$B^* = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Theta) & \sin(\Theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{Bmatrix}$$

to C. This will rotate the plane of the second and third coordinates through the angle θ with the first coordinate held fixed. One can verify easily that

$$B^*CBx = e_1 \qquad B^*\,CBy = e_2 \qquad B^*\,CBz = e_3,$$

i.e.

$$B^*CBC^T = I$$

and from this equation we may solve for B.

$$B = C^T B^{*T} C$$

Subsection (ii). <u>Assignment of Signs.</u>

One first needs the values of u, the first principal component of the 3-D displacement data swarm. We simply state that if x is a set of 3-D displacement vectors, and p is the eigenvector corresponding to the largest eigenvalue of the covariance matrice of {x}, then the values of u are the dot products

$$u = p \cdot (x - \bar{x})$$

where $\bar{x}$ is the centroid (average) of the data swarm.

When one regresses either d or Θ on u, the resulting scatter plot has the appearance of a "v", or an absolute value function. A difficulty is encountered in that the minimum of d or Θ is not necessarily (sufficiently close to) zero. This creates the need to estimate values d_0 and Θ_0 so that the replacements

$$d \leftarrow d - d_0 \qquad \Theta \leftarrow \Theta - \Theta_0$$

will produce a continuous regression scatter plot of d (resp. Θ) on u after the ordinate values on the left leg have by has been assigned negative signs. Our approach to this problem has been an expedient one: First re-index Θ,d, and u according to the rank of u (i.e. increasing order). Next select the minimum $\underline{d}(\underline{\Theta})$ of $d(\Theta)$ and let i(j) be the index of $\underline{d}(\underline{\Theta})$. Then let

$$d_0 = \text{median } (d_{i-1}, d_i, d_{i+1})$$

$$\Theta_0 = \text{median } (\Theta_{j-1}, \Theta_j, \Theta_{j+1})$$

It will be seen that this choice is modestly successful.

Should the overall transformation technique be sufficiently promising, it will be necessary to develop a more refined method to estimate d_0, Θ_0. An obvious first thought in this direction would be to fit regression lines to the two separate legs and use the intersection point. This seems attractive except in those cases for which either d_0 or Θ_0 is naturally close to zero. There some of the values in the scatter plot have already been reflected, e.g. observe the "mushy" bottom in Figure 3. This situation interferes with our ability to fit good lines.

Subsection (iii) Graphical Method to Judge the Appropriateness of the Bivariate Normal Model.

A two dimensional scatter plot that appears consistent with a straight line regression model might be studied further for distributional properties to determine whether the bivariate normal model is tenable. An important and easily exploitable property of this model is that it can be tranformed to a circular normal random vector (X,Y) and, upon transforming to polar coordinates

$$
\begin{aligned}
R^2 &= X^2 + Y^2 \\
PV &= \tan^{-1}(Y/X) \\
\Theta &= PV && \text{for } X > 0 \\
&= PV + \pi && \text{for } X < 0 \text{ and } Y > 0 \\
&= PV - \pi && \text{for } X < 0 \text{ and } Y > 0
\end{aligned}
$$

we have the exponential $(1/2\sigma^2)$ distribution to describe R^2 and the uniform $(-\pi,\pi)$ distribution to describe Θ. The two quantities are independent and the two distributions are readily verified with the use of simple plotting methods.

It seems wise to take a moment and verify these properties. The circular normal density is

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp\left\{- \frac{1}{2\sigma^2}(x^2 + y^2)\right\}$$

The inverse polar transformation is

$$x = r\cos(\Theta) \quad y = r\sin(\Theta) \quad dxdy = rdrd\theta.$$

Use of this substitution produces

$$g(r,\Theta)) = \frac{1}{2\pi\sigma^2} re^{-r^2/2\sigma^2} = g_R(r)g_{\Theta}(\Theta)$$

where $\quad g_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \; 0 < r < \infty$

and $\quad g_{\Theta}(\Theta) = \frac{1}{2\pi}, \qquad -\pi < \Theta \leqq \pi.$

The result will be completed when we make the change

$$s = r^2 \qquad ds = 2rdr$$

so that

$$g_S(s) = \frac{1}{2\sigma^2} e^{-s/2\sigma^2}, \quad 0 < s < \infty.$$

Thus Θ is distributed uniformly as $(-\pi,\pi)$

and $S = R^2$ is exponential $(1/2\sigma^2)$.

Histograms of Ⓗ and $\ell n(R^2)$ should exhibit a constant value behavior, and a scatter plot of Ⓗ on R(or R^2) should not suggest dependence.

The preliminary step of transforming the original data, say (W,Z) into potential circular normal form needs to be described:

First make the replacements

$$w \leftarrow (w - \bar{w})/S_w$$

$$z \leftarrow (z - \bar{z})/S_z$$

where $\bar{w}$ and $\bar{z}$ refer to the original center of gravity of the data and S_w and S_z are the sample standard deviations. Then compute the correlation coefficient

$$\rho = \frac{1}{N - 1} \sum_{i=1}^{N} w_i z_i$$

where N is the number of data vectors. This done, the joint distribution of W, Z is viewed as being a standard bivariate normal with correlation ρ. The final step is to apply the transformation

$$x = w$$

$$y = (z - \rho w)/\sqrt{1 - \rho^2}.$$

It is easily verified that (X,Y) has a circular normal distribution and the variance is unity.

The additional step, taking (X,Y) to (R^2,Ⓗ) was described earlier. In addition to (or in place of) the histogram plots mentioned earlier, one can apply quantile plots. i.e. theoretical quantiles plotted against empirical quantiles. The value of these plots lies in the fact that they should produce reasonable looking straight lines if the theoretical distributions are well chosen. There is great sensitivity in the tails (extremes) of these plots however.

The empirical quantiles are obtained from the order statistics:

$$\hat{Q}_{\Theta}(p_i) = \theta_{(i)}$$

$$\hat{Q}_s(p_i) = s_{(i)} = r^2_{(i)}$$

$$p_i = i/(N + 1) \qquad \text{for } i = 1,\ldots N$$

The corresponding theoretical quantiles are obtained from

$$Q_{\Theta}(p_i) = \pi(2p_i - 1)$$

$$Q_s(p_i) = -\ln(1 - p_i)$$

III. Analysis of Application.

The data sets used are those available from Reference 1; a Mark 10 track at Nanoose on 9 September 1982. The particulars are:

Data Set	Sensor Pair	Point Count Interval	No. of Usable points
ONE	4,5	3933,3999	67
TWO	4,5	4028,4097	70
THREE	4,5	4405,4458	53
FOUR	5,6	6909,7001	93
FIVE	5,6	7079,7162	82
SIX	5,6	7647,7738	87

Each of these six situations were simulated 200 times, and the displacement and orientation corrections were computed each time. Scatter plots of Θ on d appear in Figure 1 and 2. To get a reasonable view of these data, it was necessary to remove some wild simulated points. Specifically:

Data Set	ONE	TWO	THREE	FOUR	FIVE	SIX
Outliers Removed	1	24	41	0	0	0

The outlier removal rule was: Drop the values if either $d > 100$ ft or $\Theta > 0.5$ radian.

The hairpin shape is obvious for data sets one and two. It will be seen that it is also clearly present in data sets three and six as well. Data sets four and five seem to have a boundary. No interpretation for this is offered at this time. The hairpin effect may be explained as the scatter plot of the absolute values of two variables that are (otherwise) linearly related.

Some other features of Figure 1 and 2 are noted in passing. Generally the first three data sets have larger values for d and Θ than do the other three. This fact could be related to one or both of the following facts:

(i) The valid point count size is smaller for the first three; 67, 70, 53 vis a vis 93, 82, 87.

(ii) The sensor array pair is different; (4,5) vis a vis (5,6).

Figures 3 and 4 contain the scatter plots of d and Θ versus the first principal component of their respective displacement data sets. The efficacy of using a principal component to represent a multivariate data set is dependent upon the percent of total variability that is represented by that component. These figures are recorded below:

Data Set	ONE	TWO	THREE	FOUR	FIVE	SIX
Percent of Total Variance	97.0	74.6	59.3	85.5	99.4	99.1
Total Variance	6349.6	14564.2	275935.4	13.1	119.8	793.8

It is interesting to note that the two "non hairpin" cases, four and five, are of conspicuously low variability. The appearance of the "v" shape is suggestive of the absolute value of an otherwise signed variable. Since data sets 1, 2, 3, and 6 show this shape, they are the ones that can produce hairpins in the scatter plots of Θ and d. Two of these four hairpins (3 and 6) have such narrow arches that the scatter plots of the two legs do not separate (see Figures 1 and 2). We note that the plot of d vs u for Data Sets four and five is suggestive of a hairpin whose bend point is not in the center but deep in the tail. The corresponding plots of Θ vs u do not confirm the effect however.

The assignment of negative signs to the left legs of these 'V' shaped plots must be preceded by the translations.

$$d \leftarrow d - d_0 \qquad \text{and} \qquad \Theta \leftarrow \Theta_0$$

Using the local median of three technique explained in Section II, these values are

Data Set	ONE	TWO	THREE	FOUR	FIVE	SIX
D_0	2.20	6.07	15.10	15.19	9.74	15.19
θ_0	.0028	.0030	.0023	.0077	.0016	.0017

Having made these translations, negative signs are attached to all values of $d(\Theta)$ whose indices (computed from the ranks of u) are less than or equal to the index of $d,(\Theta)$ = zero (i.e. formerly $d_0,(\Theta_0)$).

This done, the Figures 1 and 2 are converted to their signed versions, Figures 5 and 6. The result appears to be reasonable in the four hairpin designated cases (Data Sets 1, 2, 3 and 6). A look at the center of the plots for Data Sets 2 and 3 suggests that the values for d_0 and Θ_0 might be better chosen. The result for Data Set 5 may not be as good as the original untransformed plot.

Turning to the application of our ad hoc test for the bivariate normal distribution, recall that the data are transformed first to the circular normal and then to polar form. Figure 7 contains the scatter plots of the squared radius vs the polar angle. It was hoped that these could show independence. The zero regression aspect of independence seems to be present, but at least three of the plots (Data Sets 1, 3, 5) exhibit some sort of cyclical effect; large radii are associated with regularly spaced polar angles.

Figures 8 and 9 contain Q - Q plots; empirical quantiles in the horizontal and theoretical quantiles in the vertical. To avoid crowding, every third point was plotted. In order to support the distributional assumptions, all of these plots should be reasonably straight lines. It is our opinion that the departures from straight lines are sufficient to cast doubt on any precise use of the bivariate normal distribution for the scatter plots in Figures 5 and 6.

FIGURE 1: ROTATION VS DISPLACEMENT USING SENSORS FOUR AND FIVE

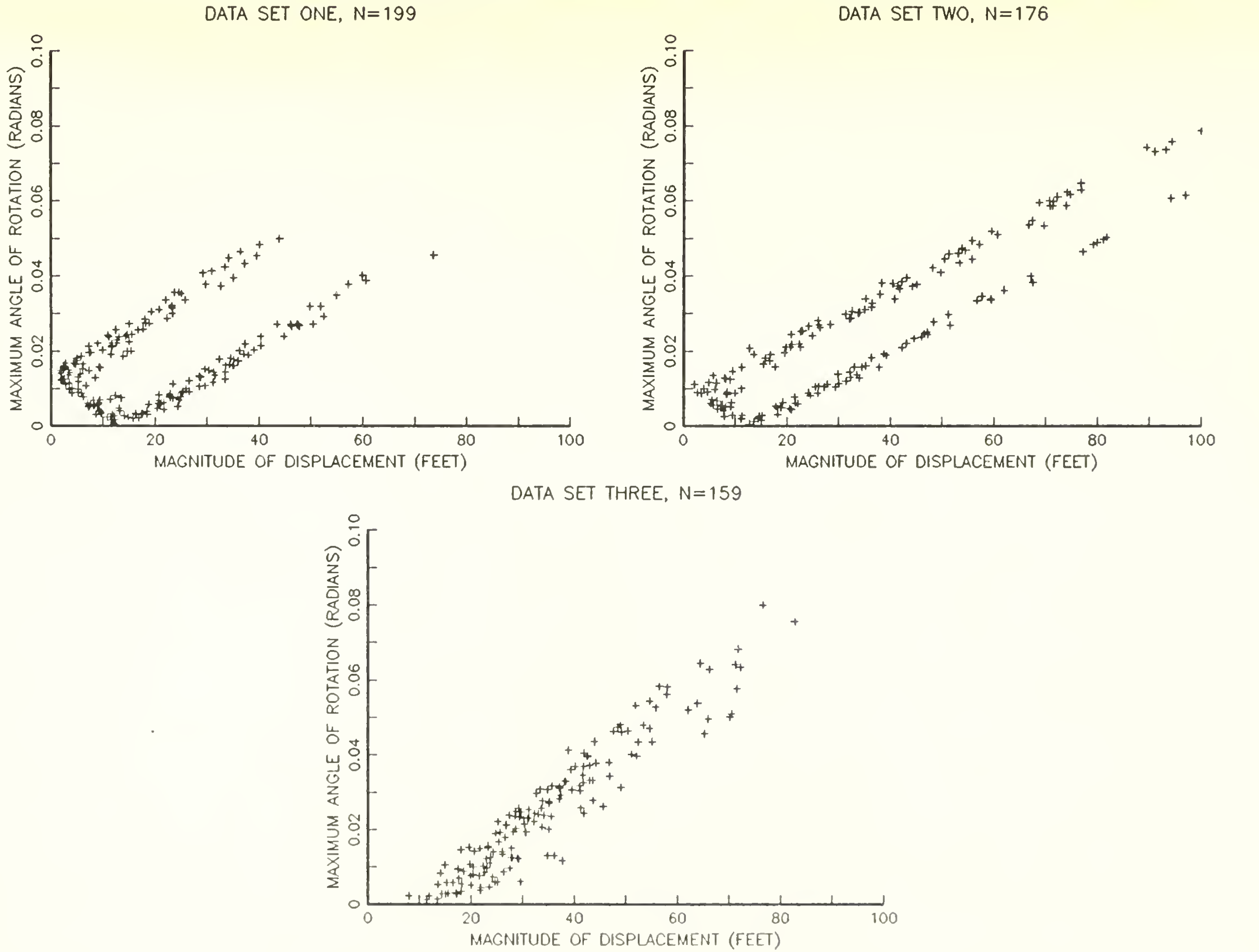


FIGURE 2: ROTATION VS DISPLACEMENT USING SENSORS FIVE AND SIX

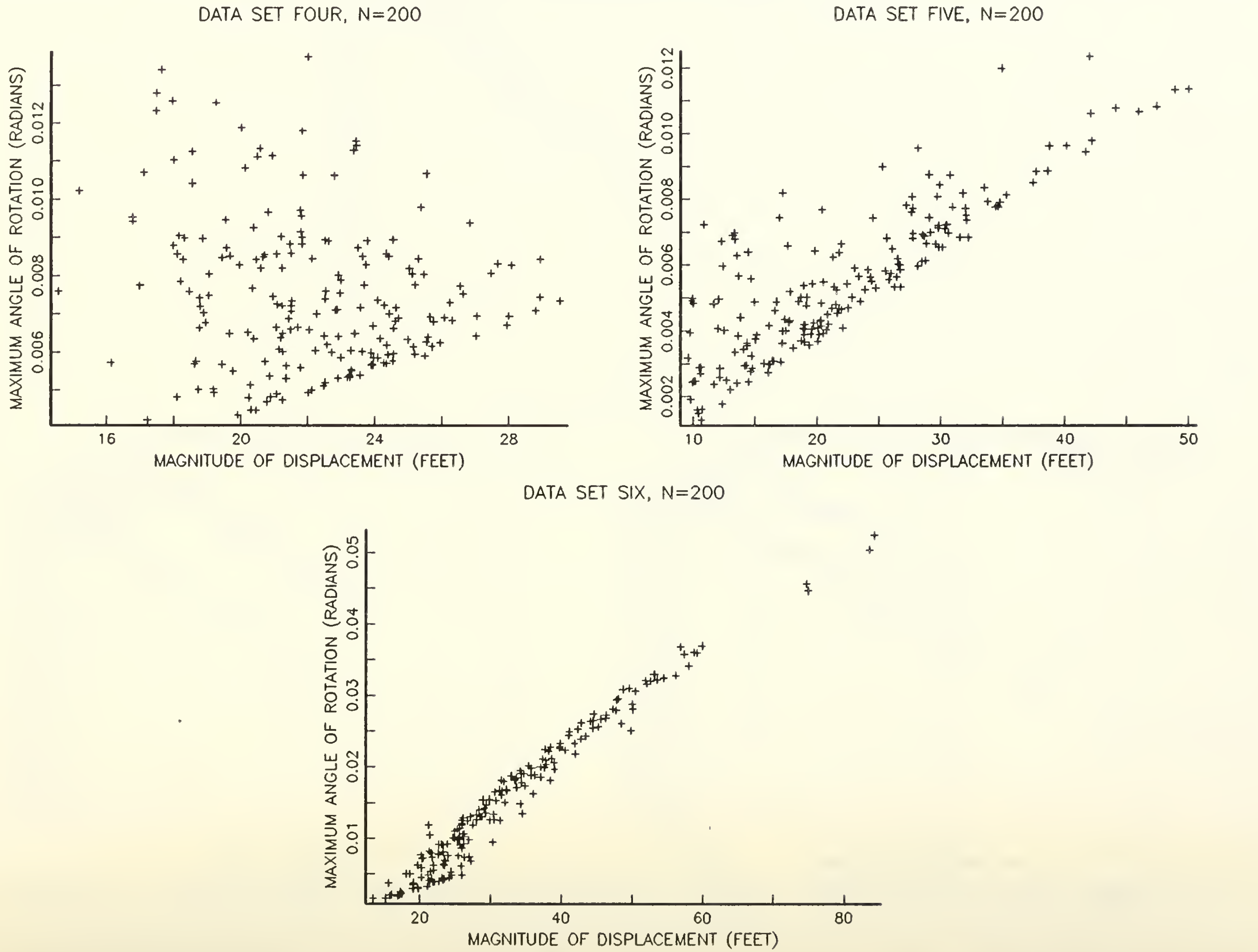


FIGURE 3: DISPLACEMENT/ROTATION VS PRINCIPLE COMPONENT

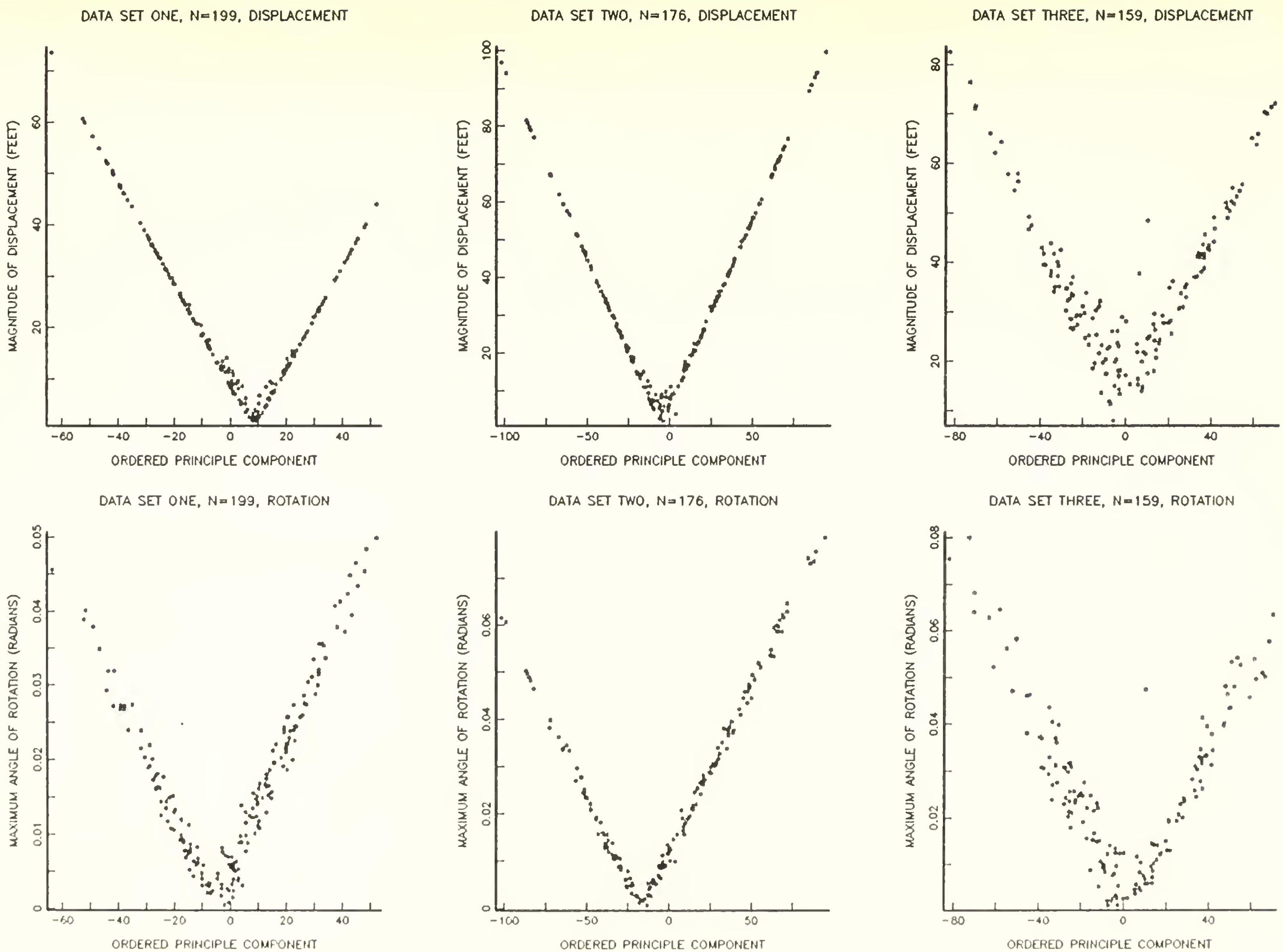


FIGURE 4: DISPLACEMENT/ROTATION VS PRINCIPLE COMPONENT

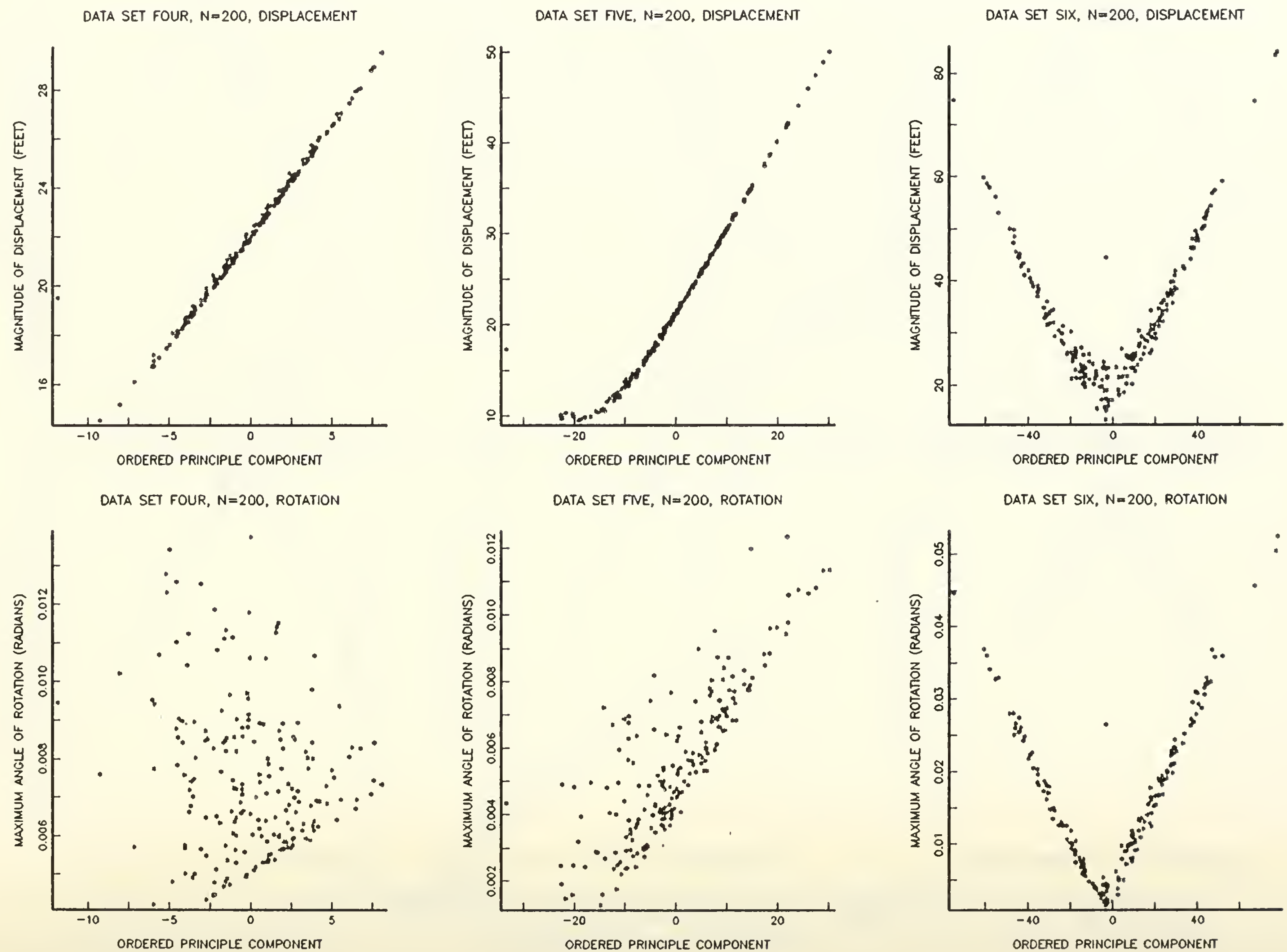


FIGURE 5: SIGNED ROTATION VS DISPLACEMENT, SENSORS FOUR AND FIVE

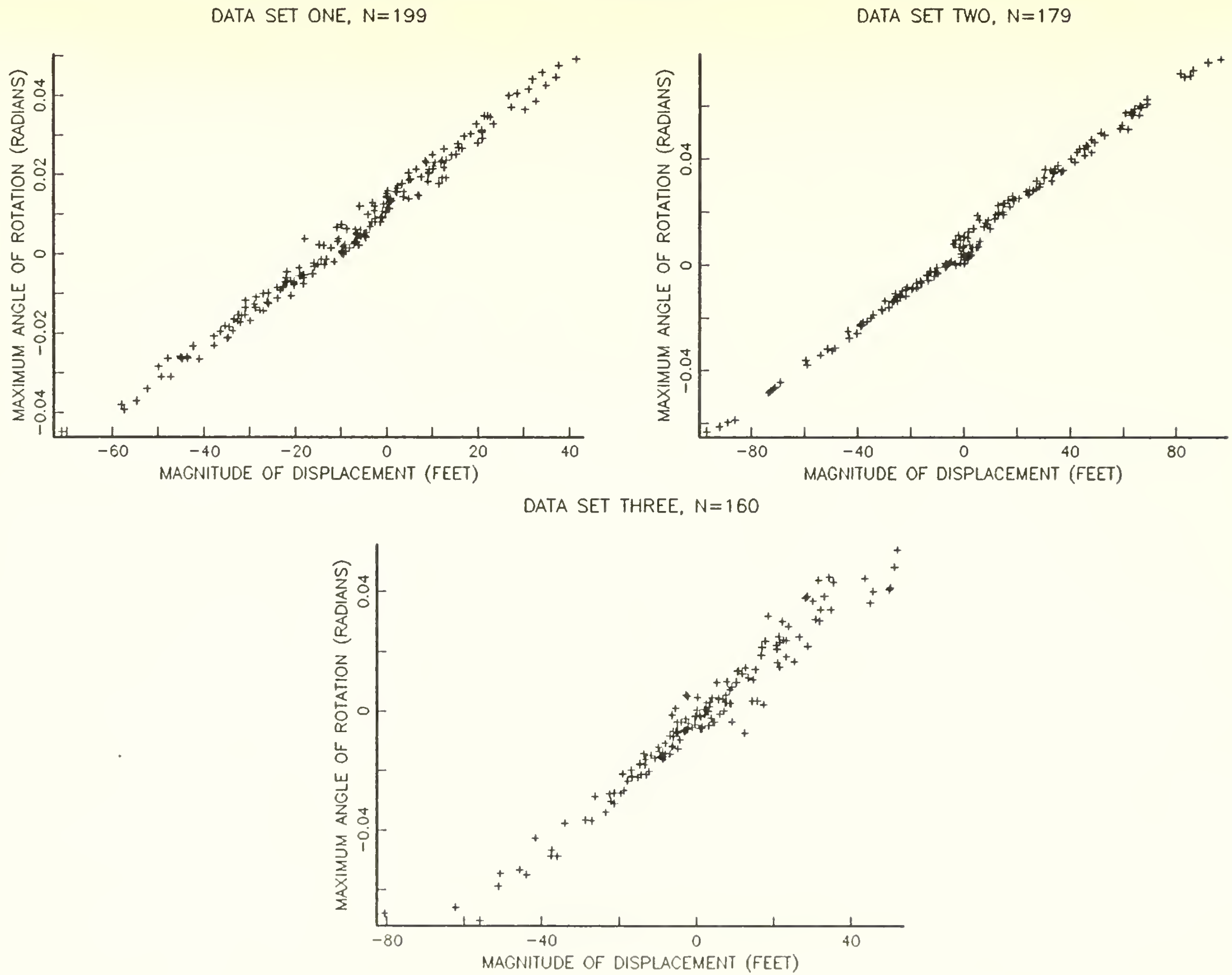


FIGURE 6: SIGNED ROTATION VS DISPLACEMENT, SENSORS FIVE AND SIX

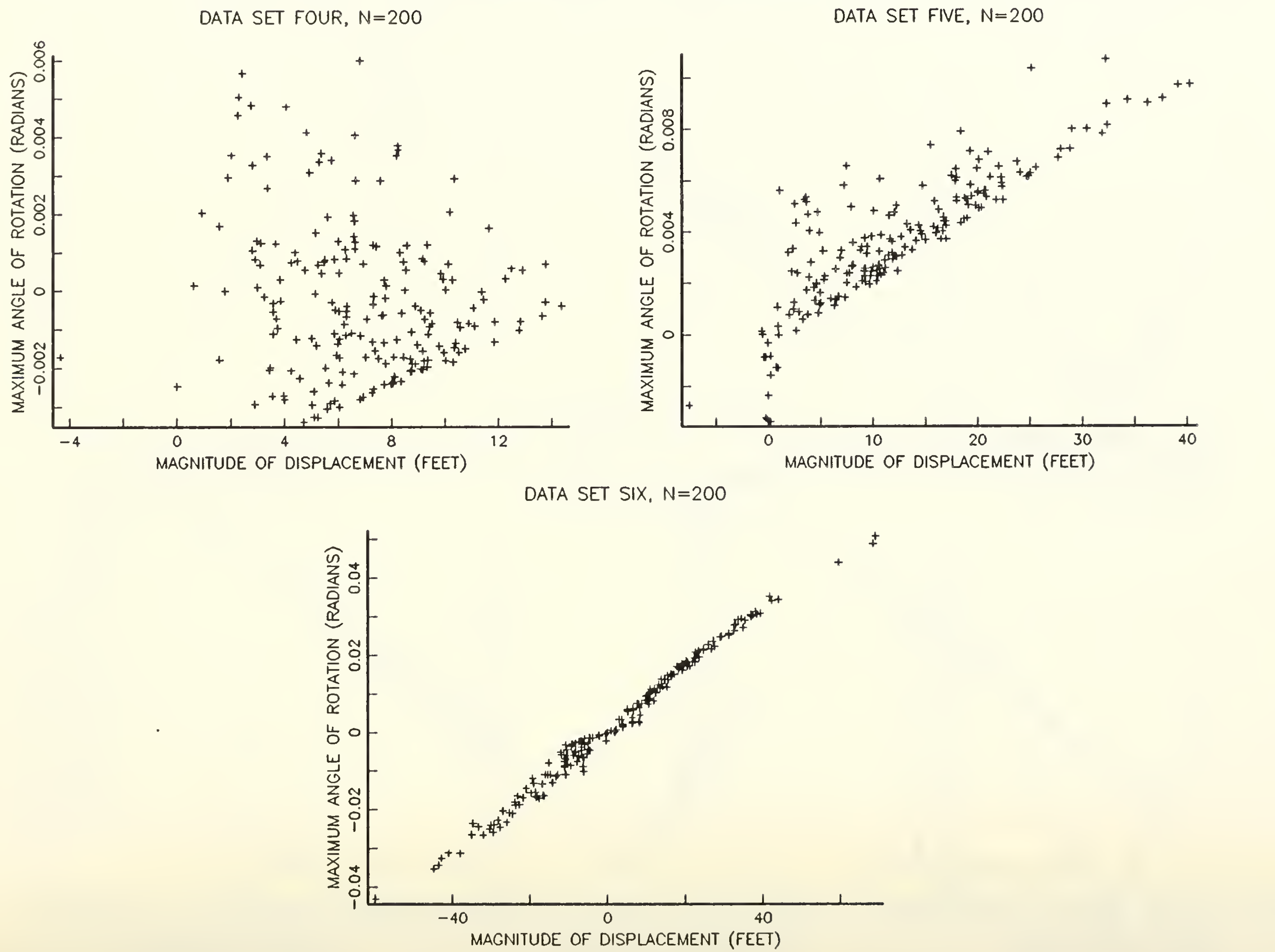


FIGURE 7: DATA SCALED AND TRANSFORMED TO POLAR COORDINATES

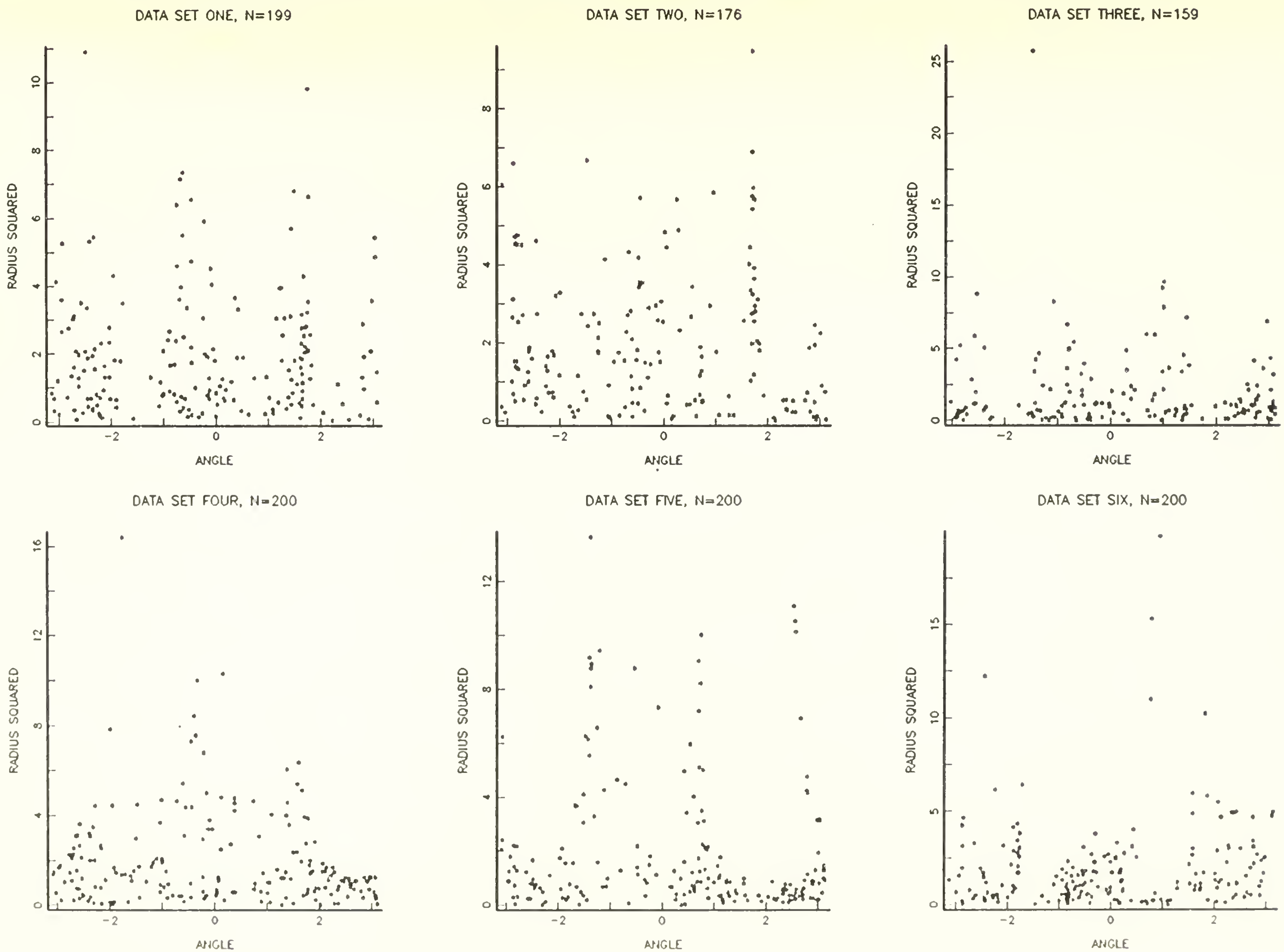


FIGURE 8: QUANTILE PLOTS, SENSORS 4 AND 5

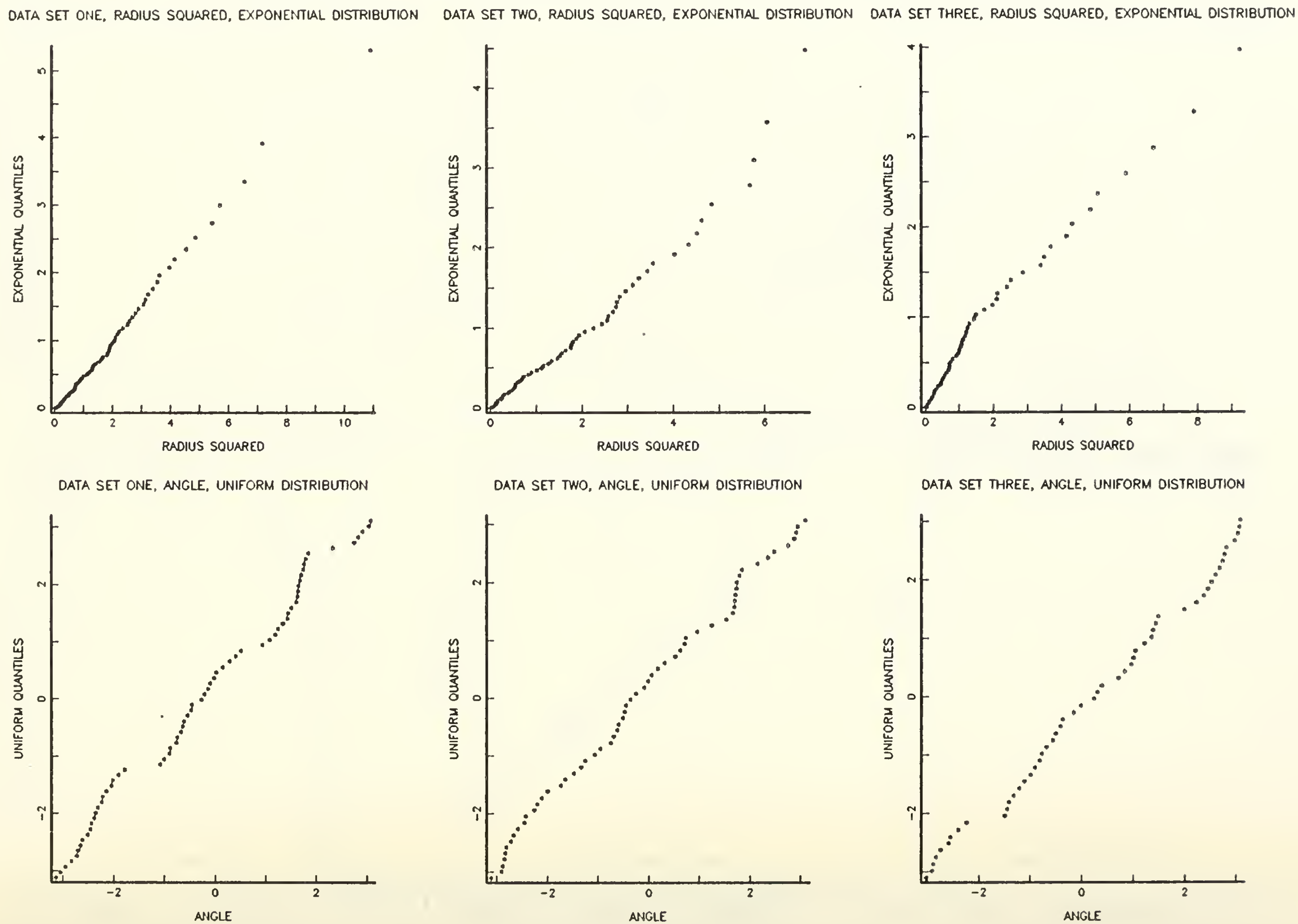
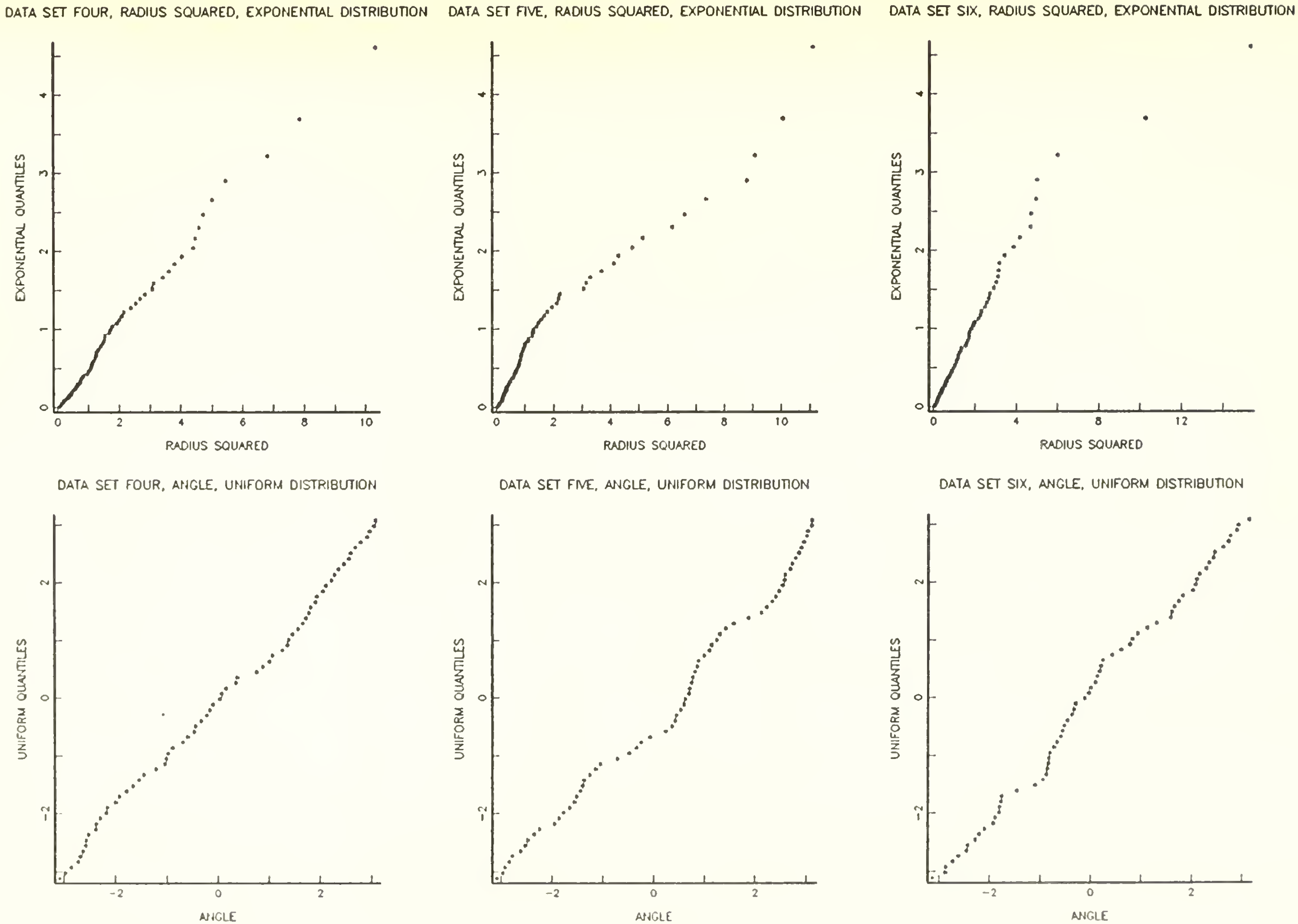


FIGURE 9: QUANTILE PLOTS, SENSORS 5 AND 6



IV. Summary

The methodology developed here is certainly useful but there are aspects of the data that are not yet understood. More and higher quality data is needed in order to continue the development. The main goal is to substitute two dimensional data for six dimensional data. In doing so, it is hoped that one can profitably use popular bivariate probability laws to describe these references distributions.

REFERENCES

1. Gygax, G., "The Simulation of Remotely Measured Paths of Underwater Vehicles for the Purpose of Monitoring the Calibration of Test Ranges," Master's Thesis, USNPS, September 1985.

2. Read, R. R., "Interim Report for NUWES, Task 83-7: Analysis of Tracking Data", USNPS, Project Report, NPS55-83-031PR, October 1983.

3. Read, R. R., "Program for the Simultaneous Estimation of Displacement and Orientation Corrections for Several Short Base Line Arrays", USNPS Technical Report, NPS55-85-028, October 1985.

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